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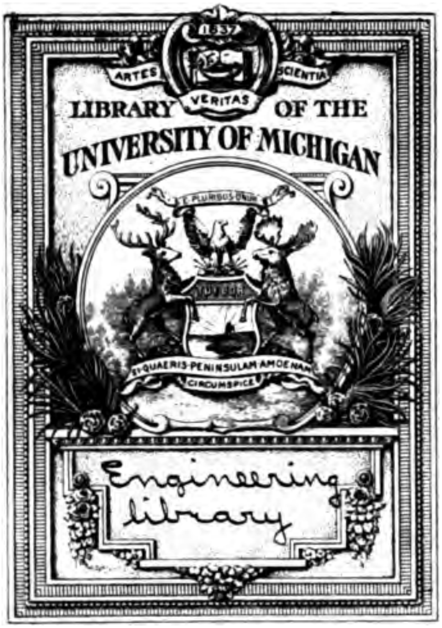

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DYNAMICS

OF

RECIPROCATING ENGINES

BY

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DYNAMICS

OF

RECIPROCATING ENGINES*

The problem of the forces which exist in reciprocating engines has many points of interest not usually considered by engineers. It is customary to consider that the pressure on the piston is transmitted unchanged, or only slightly modified by the angularity of the connecting-rod, to the crank-pin, and the moving parts are designed with reference to this pressure.

It is the purpose of this paper to consider the forces to which the crank-pin is subjected, and to show graphically the effect of the angularity of the connecting-rod and of the acceleration of the reciprocating parts, viz: the piston, piston-rod, cross-head, and the unbalanced part of the connecting-rod. The discussion will be confined to horizontal engines, and the effect of friction will be neglected as it is such a variable quantity. The application to vertical engines may be easily made when the action of the horizontal engine is fully understood. The problem will be treated by considering

First, The effort in the connecting-rod, and its effect in producing rotation of the crank.

Second, The pressure required to accelerate the reciprocating parts, and the resultant effort producing rotation of the crank.

Third, The variation in rotational effort for which the fly-wheel is provided.

First, *The Connecting-rod Effort and the Rotational Effort.*—It will be simpler to consider, first, an engine in which the connection between the piston and crank is straight, that is, a *yoke* or *slot* connection instead of a *connecting-rod*. Later the first will be spoken of as an *infinite* connecting-rod and the later as a *finite* connecting-rod.

With the yoke connection the pressures on the piston are transmitted in parallel lines for all positions of the crank.

(* Revised from TECHNIC, 1888.)

In Fig. 1, let O be the center of the circle described in the crank-pin C, and OC any position of the crank making an angle θ with the line of the centres. The diameter of the circle, $A'B'$ is equal to, and for our purpose will represent, the stroke of the piston. Let P_i = the effort on the piston acting on the crank-pin in the line YC. P_i may be resolved into two forces, RC acting radially, and CT acting tangentially. The second produces rotation, and will be referred to as the *tangential effort;* the first produces friction only and is lost so far as turning the engine is concerned.

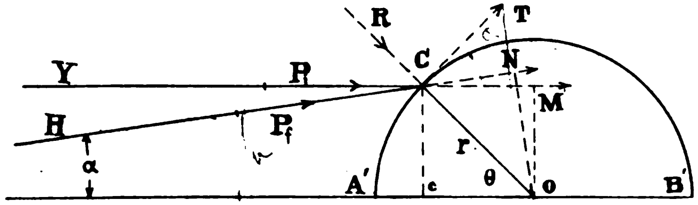

FIG. 1.

If P_i is taken to a scale, then the values of tangential and radial efforts may be measured directly.

It is more convenient, however, to determine the tangential effort by taking from the center O a perpendicular OM upon the line of action of the force P_i produced.

Denoting the tangential effort by T_i and the radius of the circle by r, and taking moments about O, we have

$$T_i \cdot r = P_i \cdot \overline{OM} \ldots\ldots (1).$$

$$\text{whence } T_i = \frac{P_i \cdot \overline{OM}}{r} \ldots\ldots (2).$$

If r be taken to a scale = P_i, then $T_i = OM$. Taking Cc perpendicular to $A'B'$, we have $T_i = OM = Cc = r$ sine θ......(3); or the tangential effort is represented by the sine of tne crank angle, and is equal to it when $P_i = r$.

If now the case of a connecting-rod is considered, let HC represent the line of action of the force, making an angle a and with the line of centres. Produce HC and from O drop upon it the perpenlar ON. Let P_f and T_f denote the connecting-rod and the tangential efforts respectively; then as before

$$T_f \cdot r = P_f \cdot \overline{ON} \ldots\ldots (4).$$

$$\text{whence } T_f = \frac{P_f \cdot \overline{ON}}{r} \ldots\ldots (5).$$

and if r be taken = P_f, then $T_f = \overline{ON}$.

As will be seen the connecting-rod has the effect of increasing the tangential efforts in the first and fourth quadrants, and of decreasing these efforts in the second and third quadrants.

The value of P_t may be found by multiplying by the secant of the angle a, or $P_t = P_i \sec a$......(6).

For usual ratios of connecting-rod length to crank the value of P_t is from 1.015 P_i to 1.03 P_i when the connecting-rod and crank are at right angles, being of course equal to P_i at the ends of the stroke.

It is unimportant to consider this difference in practice except when the connecting-rod is short relatively to the crank. It may then be useful to know the efforts in the rod for each position of the crank, and these efforts may be found graphically as follows:

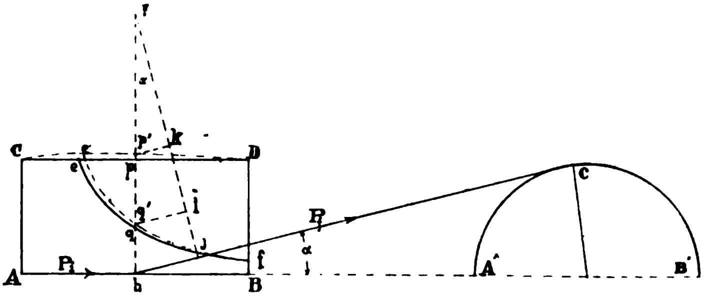

FIG. 2.

In Fig. 2 let $A\ B\ C\ D$ represent an indicator diagram from an engine in which the steam follows full stroke, and $A\ B\ C\ e\ f$, one from an engine cutting off at e.

Let h be the position of the piston, $h\ c$ the connecting-rod and c the crank-pin, describing the crank circle on the diameter $A'\ B'$. The ordinate $h\ p$ represents the pressure against the piston for the assumed position. Produce $h\ p$ and erect $j\ I$ perpendicular to the connecting-rod, intersecting $h\ p$ produced in I. Take $j\ k = h\ p$ and draw $k\ p'$ parallel to $h\ c$; then $h\ p'$ will represent, to the same scale as $h\ p$, the effect in the connecting-rod. For, taking moments about I we have

$$P_i \cdot Ih = P_t \cdot Ij, \text{ whence } P_t = P_i \cdot \frac{Ih}{Ij} \ldots.(7):$$

$$\text{by similar triangles } \frac{Ih}{Ij} = \frac{hp'}{Jk} = \frac{hp'}{hp}; \text{ or, } P_t = P_i \frac{hp'}{hp} \ldots.(8).$$

But $P_i = hp$, therefore $P_t = hp'$. In the same way for the pressure hq against the piston, the pressure hq' exists in the connecting-rod. The dotted diagrams represent the indicator diagrams as modified by

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the connecting-rod. As before stated this calculation may be neglected except in extreme cases.

Having now a simple method of finding the tangential efforts for each position of the crank and corresponding position of the piston, it becomes easy to show by a diagram the tangential efforts for an entire revolution.*

In Fig. 3 let the upper part represent indicator diagrams in which $A\ B$ is the stroke, $A\ C\ D\ B\ A$, a diagram following full stroke, and $A\ C\ E\ F\ B\ A$, a diagram following part stroke. Consider first the diagrams shown by full lines. Divide the diagram into the usual ten equal spaces and erect in the middle of the spaces the ordinates numbered from 1 to 10. Each ordinate will represent the pressure against the piston for the space in which it stands, and the average of these ordinates will of course represent the average pressure against the piston for an entire stroke. On the diameter $A'\ B' = A\ B$, describe the crank circle, and project the ordinates as shown, cutting $A'\ B'$, also the circumference, in the points numbered from 1 to 10. Through these latter points of division draw the radial lines $s_1\ t_1$, $s_2\ t_2 \ldots\ s_{10}\ t_{10}$. With a radias $Os = A\ C =$ the uniform pressure for the stroke diagram, describe the semi-circle $s_1\ s_2 \ldots\ s_{10}$. This last circle is called the *radial diagram of pressures* for full stroke. It amounts, of course, to taking the ordinates from the indicator diagram and laying them off from the centre O on the radial lines which, to avoid confusion, have not been drawn from the centre to the circle $s_1\ s_2$ &c.; that is, Os_1, Os_2 &c., are taken equal to ordinates 1, 2 &c., of the indicator diagram.

If from the points $s_1\ s_2\ s_3$ &c., perpendiculars be dropped upon the line of centres $A'\ B'$, these perpendiculars will, as already shown, represent the tangential efforts for the corresponding crank positions with an indefinite connecting-rod.

Now take these perpendiculars and lay them off outside the crank circle, considered as a base circle; that is, take $1t_1$, $2t_2$, $3t_3 \ldots 10t_{10}$, equal respectively to the perpendiculars upon $A'\ B'$ from s_1, $s_2 \ldots s_{10}$, and connect the extremities by the curve $A'\ t_1\ t_2 \ldots t_{10}\ B'$: this line is known as the *curve of tangential efforts*, and shows at a glance the turning effort for all positions of the crank from the beginning to the end of one stroke. The curve for the return stroke with an infinite connecting-rod [not shown] would be identical with that for the forward stroke.

Considering the diagram in which steam follows with full pressure through CE and expands through EF, the radial diagram becomes $s_1\ s_2\ s_3\ s_4\ s'_5 \ldots s'_6 \ldots s'_{10}$, and the sines or perpendiculars being drawn and laid off as before give the tangential diagram $A'\ t_1\ t_2\ t_3\ t_4\ t'_5\ t'_6 \ldots t_{10}\ B'$.

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* The radical and tangential diagrams used in this paper were originally presented by Mr. Arthur Rigg, in England, in 1870. See "A Practical Treatise on the Steam," by Rigg.

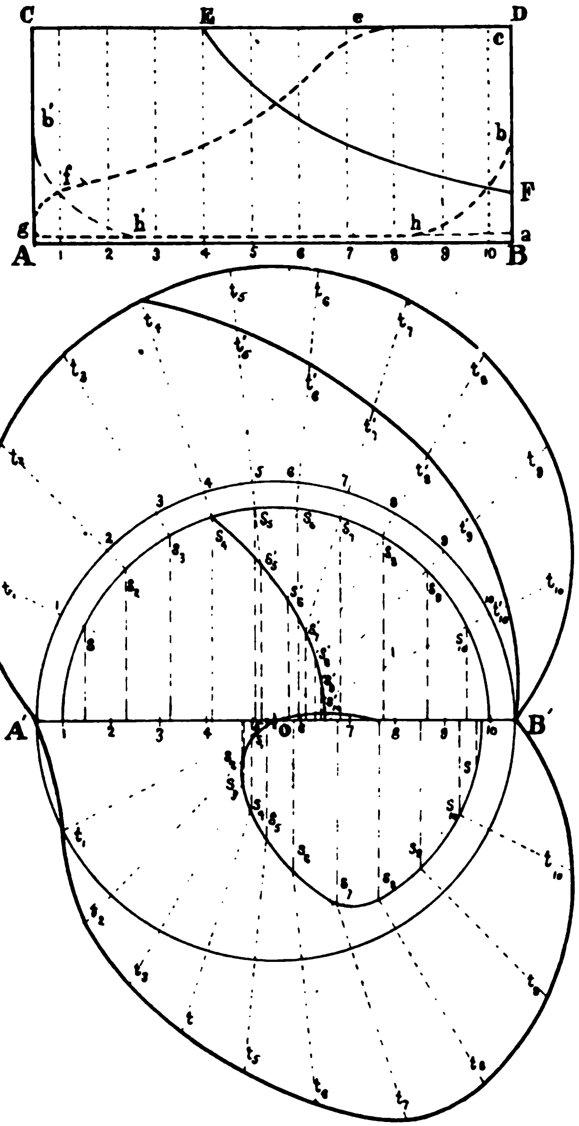
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FIG. 3.

The indicator diagrams thus far considered are ideal, and are never fully realized in practice. The dotted diagram shown for the return stroke is modified to represent more closely actual practice, the corners at e, f, g and hb being rounded for practical reasons; the back-pressure line ga lies above AB by an amount equal to the back-pressure. It will be evident that as steam pushes the pisten on one side with a pressure varying, as shown by the line $c\ e\ f\ g$, the piston pushes or expels the steam from the cylinder on the other side with a pressure varying, as shown by the line $a\ h\ h'\ b'$; therefore, the *useful* pressure will be the difference between the ordinates of these lines, becoming negative where they cross at f. In the lower half of the crank circle, then, construct the radial diagram $s_{10}\ s_9 \ldots s_1$ by taking the difference of these pressures. Where the expansion line eg crosses the compression line $h'b'$, the pressures on the two sides of the piston are equal, and the radial diagram passes through the center O. From this point to the end of the stroke the back pressure is greater than the forward pressure, and the radial diagram continues as shown.

By erecting the perpendiculars and laying off outside of the base circle, as before, we get the tangential diagram $B'\ t_{10}\ t_9 \ldots t_1\ A'$. Between t_2 and t_1 it passes inside the base circle, showing that instead of a rotational effort driving the crank, the crank drives or pulls the piston. The diagram, then, shows at a glance the effect of efforts on the piston in turning the crank; and all being constructed to a scale, quantities may be measured with as much accuracy as the original indicator diagrams.

If preferred a straight line may be used instead of the base circle, the tangential efforts being laid off on ordinates spaced the same as on the circle, at right angles to the base line.

The *mean* tangential effort may be found by dividing the tangential diagram into equal spaces on the base circle, drawing radial ordinates, and averaging the same as the indicator diagram; or it may be found as follows: The useful work done on the piston is of course equal to the useful work done on the crank-pin. While the piston travels the length of its stroke $A\,B$, the crank-pin travels the length of the semicircle on $A'\ B'$.

$$\text{This latter length} = A'\ B'\,\frac{\pi}{2} = A\ B\,\frac{\pi}{2} = 1.5708\ A\ B \ldots (9).$$

Let p equal the mean effective pressure per sq. inch against the piston for a stroke, found by averaging the ordinates of the indicator diagram and multiplying by the scale of the diagram, and t, the tangential effort corresponding to p, then

$$p \cdot \overline{A\ B} = 1.57\ \overline{A\ B} \cdot t \text{ whence } t = \frac{p}{1.57} \ldots (10).$$

The *total* mean effective pressure and the *total* mean tangential effort will of course be found by multiplying p and t by the area of the piston.

The value of t may be laid off from the base circle to the proper scale and a circle drawn through it from centre O; this circle is called the *circle of mean tangential effort.* Figs. 5, 14 and 16 show these circles. The variations outside and inside of this circle should be small to give uniform speed with a light fly-wheel.

In Fig. 4 the tangential diagrams of Fig. 3 are reproduced in dot and dash line and the full line shows the effect of considering the

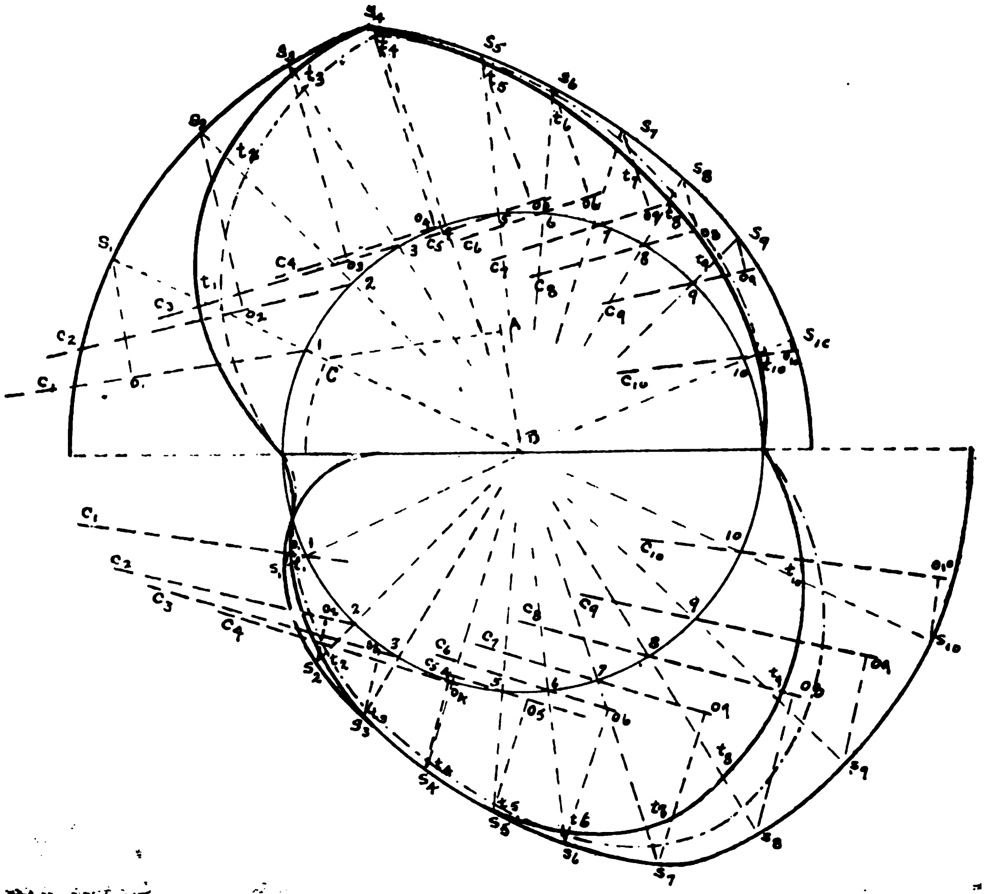


FIG. 4.

angularity of the connecting-rod. The construction is as follows: $C_1 1$, $C_2 2$, $C_3 3$............$C_{10} 10$ are positions of the connecting-rod corresponding to the crank positions *1*, *2*, *3*....*10* C represents a point on the radial diagram as in Fig. 3; through C draw CA parallel to the corresponding crank position, from the center B let fall a perpendicular on CA. This perpendicular, as already shown, will be equal to the tangential efforts. The following construction gives less confusion; from the point *1* draw the radius $1S$, equal to the pressure of the piston and equal to CB. From the point S let fall perpendicular on the connecting-rod position C_1 *1* cutting it in O_1. Now since the two triangles ACB and $1S_1O_1$ have two sides parallel and one side equal, the other sides are equal, hence AB equals S_1O_1 equals the tangential effort. In a similar manner we find the tangential efforts S_2O_2, S_3O_3, S_4O_4............$S_{10}O_{10}$. These tangential efforts are laid off from the base circle giving the curve t_1, t_2, t_3....t_{10}. In the diagrams that follow the piston pressures have been laid off from the centers of the crank circles. The same construction follows for the return stroke shown in the lower half of the circle.

Thus far only a single engine has been considered; but the same construction may be applied to an engine with any number of cylinders, care being taken to reduce all of the diagrams to the same scale before combining, considering not only the scale of the indicator diagrams, but also the relative sizes of the cylinders. For example, take a two-cylinder compound engine with the condensing cylinder twice the volume of the non-condensing cylinder. Let the scale of the indicator diagram from the first be 20 lbs. to the inch, and from the latter, 60 lbs. to the inch; then the tangential efforts of the non-condensing cylinder will need to be multipled by the fraction 3-2 in order to reduce both to the condensing cylinder. Thus the area of condensing cylinder × combined tangential ordinate will equal total turning effort. This may be conveniently done by the use of proportional dividers. In referring the tangential diagram to a common base circle preparatory to combining, care should be taken that no error be made in the relative positions of the diagrams.

Fig. 5 shows a pair of diagrams worked up for the U. S. S. Quinnebaug.

These engines are two-cylinder compound, the dimensions being 42 ins. and 64 ins. diameter, and 42 ins. stroke. The condensing cylinder is 90° ahead of, or 270° behind, the non-condensing cylinder. The diagrams represent the engines doing 1102 I. H. P., or 590 for the non-condensing, and 512 for the condensing cylinder; 61 revolutions were made per minute, with a boiler pressure of 78 lbs. per sq. in., and a vacuum of 19 ins. The scales of the diagrams were 40 and 20 lbs. to the inch, but as they have been reduced to two-thirds full size, the scales have been marked 60 and 30 lbs. to the inch. The circular and tangential diagrams were made separately and then combined, as shown in the figure. Light full lines indicate the non-condensing, and dotted lines, the condensing cylinder diagrams, the heavy full line the combined diagram. The circles of mean tangen-

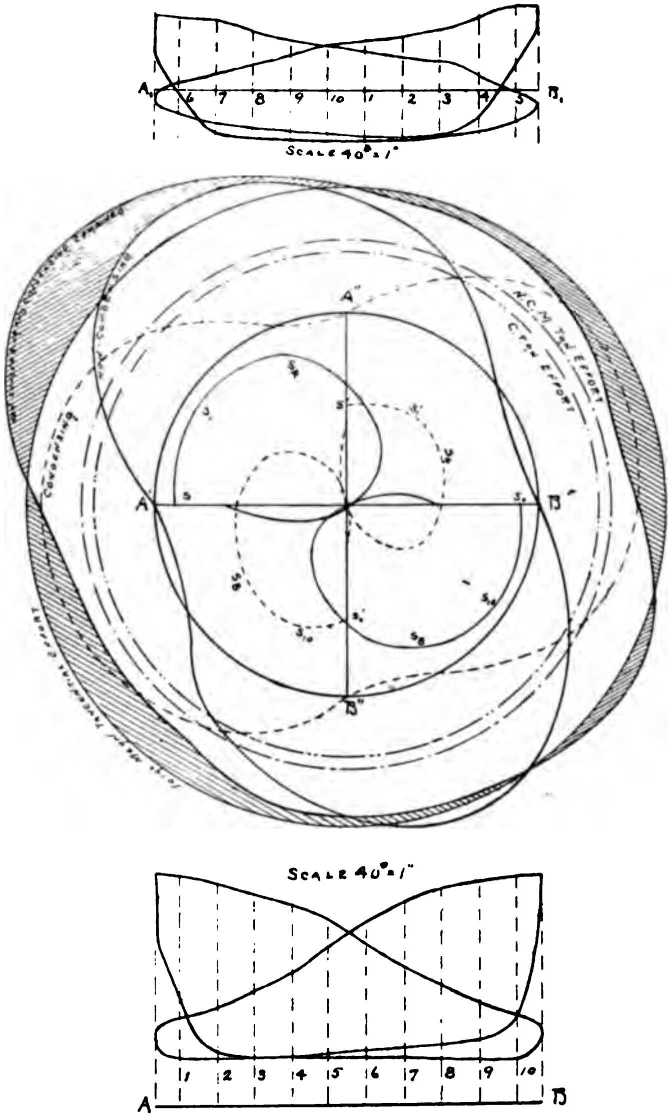


FIG. 5.

tial efforts are marked as shown. The hatched portions outside and inside of the combined mean tangential effort circle show the variation of the tangential efforts. The diagram will be clear without further explanation, and it will be apparent that the maximum tangential effort for which the shaft and crank must be designed may be easily located, and measured with accuracy. Frequently, surprising irregularities are found by the use of the diagrams as explained; but more surprising will the irregularities be found when the effect of acceleration is considered.

Second, The Efforts as Affected by Acceleration of the Reciprocating Parts.—Thus far only the tangential efforts have been considered, and these unaffected by disturbing influences other than those due to the angularity of the connecting-rod. We will next consider the effects of acceleration and retardation of the reciprocating parts.

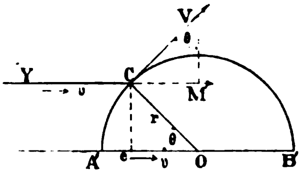


FIG. 6.

As before, first consider an engine with an infinite connecting-rod, and in Fig. 6 let O be the center of a crank circle described on $A'B'$ as a diameter. Take the crank OC, making any angle θ with $A'B'$, and draw YC and CV to represent direction of piston and tangential efforts respectively. Erect the perpendiculars VM and Cc. Let V = velocity of crank-pin, and v = velocity of piston. This is, then, a case of simple 'harmonic motion. If $V = CV$ = uniform tangential velocity of crank-pin, $CM = V \sin \theta = v$ = variable velocity of piston; and if r be taken equal to V, then $Cc = CM = v$. If $v = CM = V \sin \theta$ = velocity of piston,

$\frac{dv}{dt} = V \cos \theta \frac{d\theta}{dt}$ = the acceleration. But $Vt = r\theta$, and $\frac{d\theta}{dt} = \frac{V}{r}$,

whence $\frac{dv}{dt} = \frac{V^2}{r} \cos \theta = y \ldots . (11)$.

From the figure $cO = r \cos \theta$ whence $\cos \theta = \frac{cO}{r} = \frac{r - A'c}{r} = \frac{r-x}{r}$,

where x = distance piston has moved from beginning of stroke.

Therefore, $y = \frac{V^2}{r^2}(r-x) \ldots . (12)$, the equation of a straight line.

For $x = O$, $y = \frac{V^2}{r}$, for $x = r$, $y = O$, for $x = 2r$, $y = -\frac{V^2}{r}$, showing that this line of acceleration at the beginning of the stroke is above, at the centre crosses, and at the end is below, the horizontal, as shown in the lower half of Fig. 7. The values of the acceleration may be found graphically as follows:

In Fig. 7, take the circle with diameter divided into equal spaces, and numbered as before. Erect ordinates cutting circle in points ${}_1c\ {}_2c \ldots c_{10}$, and from these points draw the horizontals $c_1\,p_1$, $c_2\,p_2 \ldots \ldots c_{10}\,p_{10}$. These horizontals, or cosines, will be *proportional* to the accelerations corresponding to the different crank positions; if the velocity of the crank-pin be taken equal to the radius, then these cosines will be *equal* to the accelerations, in which case the lower half of the figure may be constructed by transferring with the dividers these cosines, and setting them off from the line $A'\ B'$ as shown. The line $a\ b$ is the *acceleration line.*

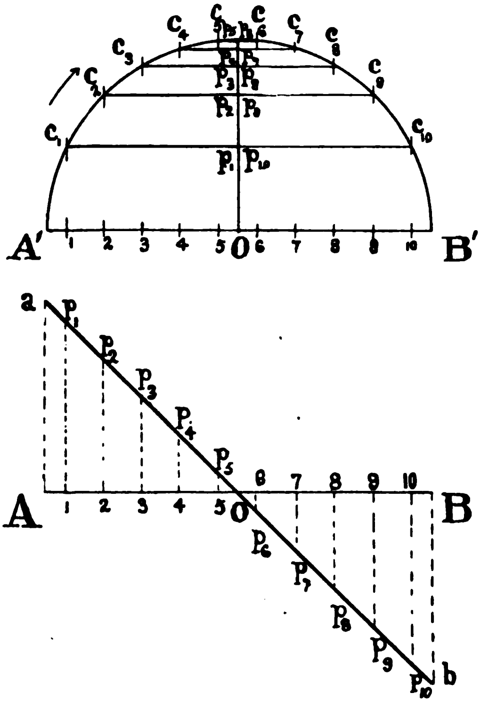


FIG. 7.

$\frac{V^2}{r}$ is the expression for the centrifugal force of unit mass, and $\frac{W}{g} \cdot \frac{V^2}{r}$ is the expression for centrifugal force of a body of weight W, which in our case represents the weight of the reciprocating parts. In practice it is more convenient to reduce this weight to the equivalent weight per square inch of piston area, or $w = \frac{W}{A}$, where A = area of piston. In ordinary cases the value of w varies from 2.5 lbs. to 5 or 6 lbs. As an example take an engine 10 ins. diameter, and 12 ins. stroke, making 300 revolutions per minute, with a value of $w = 3$ lbs.

$$V = \frac{300 \times \pi}{60} = 15.708 \text{ ft. per second, and } y = \frac{3 \times \overline{15.7}^2}{32.2 \times \frac{1}{2}} = 45.9 \text{ lbs., or a}$$

pressure of 45.9 lbs. per sq. inch is required to start the reciprocating parts from rest, assuming no friction; and an equivalent pressure is given out at the end of the stroke.

If $w = 2$ lbs., the pressure for acceleration becomes $\frac{2}{3} \times 45.9 = 30.6$ lbs. per sq. inch. If the revolutions are 200, the pressure becomes $(\frac{200}{300})^2 \times 45.9 = 20.4$ lbs. per sq. inch, and with 100 revolutions the pressure becomes $(\frac{100}{300})^2 \times 45.9 = 5.1$ lbs. per sq. inch.

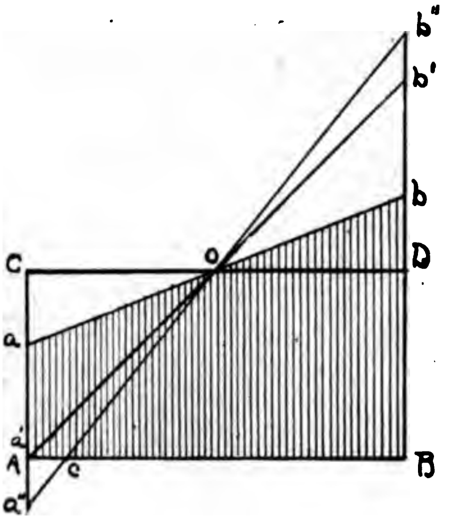


FIG. 8.

In Fig. 8 let $A\,C\,D\,B\,A$ be indicator diagram following full stroke. Take $C\,a$ equal to the pressure required for acceleration at the beginning of the stroke $= D\,b$ the equivalent pressure due to retardation at the end of the stroke. Draw $a\,b$ cutting $C\,D$ in O. Then the effective pressure against the piston at the beginning of the stroke is $A\,C - C\,a = A\,a$, and at the end of the stroke, $B\,D + D\,b = B\,b$, while at the middle of the stroke it remains unchanged. During the first half of the stroke work represented by the triangle $C\,O\,a$ has been done on the reciprocating parts by accelerating them, and during the second half, work represented by the equal triangle $D\,O\,b$ has been done *by* the reciprocating parts, in retarding them the *total* work remaining unchanged.

With a higher speed let $C\,a' = D\,b'$ = the pressure for acceleration and retardation. Then at the beginning of the stroke there is no effective pressure against the piston, and at the end the pressure is twice as great as the indicator diagram shows. Any increase of speed beyond this point gives a negative pressure at the beginning of the stroke, as shown by the line $a'' b''$. From A to c there is a pull on the reciprocating parts by the crank, this pull changing to a push by the steam after passing c. Attention need now scarcely be drawn to the fact that the pressure for which the moving parts of engines are usually designed may be far from the correct pressure. It is also evident that in general the reciprocating parts should be made as light as is consistent with strength, and the number of revolutions should not be increased unduly. An engine that runs quietly and smoothly at one speed may be noisy and give trouble at a different speed.

With slow speed engines, and with engines having relatively long connecting-rods, the solution given is sufficiently exact for practical purposes; but with high speed engines, and with engines having relatively short connecting-rods, the following solution becomes necessary.

Consider an engine with a finite connecting-rod

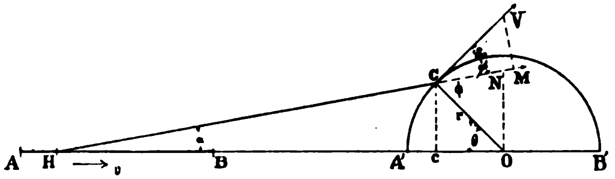


FIG. 9.

and in Fig. 9 take the crank circle as before, and draw the connecting-rod HC, producing it to M. Draw VM perpendicular to HM, and cC and ON perpendicular to $A' B'$. Let $A B$ be the stroke of the engine, and let the piston be at H. Let $V = CV$ = the tangential velocity of the crank-pin, and v = velocity of piston or cross-head, a = angle made by connecting-rod with line of centres, Θ = crank angle, $\Phi = OCM = CVM$. Denote the lengths of connecting-rod and crank by l and r respectively.

While the crank has moved from A' to C through the angle Θ, the piston has moved from A to H. The ratio of the velocity of the piston v to the velocity of the crank-pin V may be found by projecting each velocity upon the dirction of the connecting-rod, which, being a rigid body, must contain equal components of the velocities in the direction of its length. Therfore

$$v \cos a = V \sin \Phi \ldots (13).$$

$$HC \sin a = l \sin a = r \sin \Theta,$$

Whence, $\sin a = \frac{r}{l} \sin \Theta$, and $\cos a = \sqrt{1 - \frac{r^2}{l^2} \sin^2 \Theta} \ldots (14).$

But $\Phi = a + \theta$, therefore, $\sin \Phi = \sin a \cos \theta + \cos a \sin \theta$....(*15*).
Substituting the values of $\sin a$ and $\cos a$ from (*14*), we have

$$\sin \Phi = \frac{r}{l} \sin \theta \cos \theta + \sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta} \,.\, \sin \theta \text{....(16)}.$$

Substituting in (*13*) and dividing by (*14*),

$$v = V \left\{ \frac{\frac{r}{l} \sin \theta \cos \theta + \sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta} \,.\, \sin \theta}{\sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta}} \right\} \text{....(17)}.$$

$$= V \left\{ \sin \theta + \frac{\sin \theta \cos \theta}{\sqrt{\frac{l^2}{r^2} - \sin^2 \theta}} \right\} \text{....(18)}.$$

Comparing this with the equation for the relative velocities with an infinite connecting-rod, $v = V \sin \theta$, we see that the additional effect of a finite rod is determined by the fractional term which is expressed with only the crank angle as a variable.

For the acceleration we have

$$y = \frac{dv}{dt} = V \left\{ \cos \theta + \frac{\left\{ \frac{l^2}{r^2} - \sin^2 \theta \right\} \cos 2\theta - \sin^2 \theta \cos^2 \theta}{\left\{ \frac{l^2}{r^2} - \sin^2 \theta \right\}^{\frac{3}{2}}} \right\} \frac{d\theta}{dt} \text{....(19)}.$$

But $Vt = r\theta$, and $\frac{d\theta}{dt} = \frac{V}{r}$; whence

$$y = \frac{V^2}{r} \left\{ \cos \theta + \frac{\left\{ \frac{l^2}{r^2} - \sin^2 \theta \right\} \cos 2\theta - \sin^2 \theta \cos^2 \theta}{\left\{ \frac{l^2}{r^2} - \sin^2 \theta \right\}^{\frac{3}{2}}} \right\} \text{....(20)}.$$

An expression differing from that secured for the acceleration with an infinite rod by the fractional term alone.

$$\text{For } \theta = 0^\circ,\ y = \frac{V^2}{r}\left\{1 + \frac{r}{l}\right\};$$

$$\theta = 90^\circ,\ y = -\frac{V^2}{r}\,\frac{1}{\sqrt{\frac{l^2}{r^2} - 1}};$$

$$\theta = 180^\circ,\ y = -\frac{V^2}{r}\left\{1 - \frac{r}{l}\right\}.$$

$$\text{Assuming } \frac{l}{r} = 5,\ \text{then for } \theta = 0^\circ,\ y = \frac{V^2}{r}\left\{1 + \frac{1}{5}\right\} = 1.2\,\frac{V^2}{r};$$

$$\theta = 90,\ y = -\frac{V^2}{r}\,\frac{1}{\sqrt{24}} = -\ .204\ \frac{V^2}{r};$$

$$\theta = 180,\ y = -\frac{V^2}{r}\left\{1 - \frac{1}{5}\right\} = -\ .8\ \frac{V^2}{r}.$$

Solving for other values of θ we have values of y which when plotted give the curve $a\ b$, shown in the lower half of Fig. 10. The value of Aa is ⅕ larger, and that of Bb ⅕ smaller than with the infinite rod, while the acceleration line crosses $A\ B$ at a point distant from O bv very nearly ⅕ of the acceleration for an infinite rod. For the return stroke we have the line $b'\ a'$.

The following graphical solution of the equation will be found very convenient, and nearly exact:

In Fig. 10 describe the circle with radius $O\ A'$ = the acceleration effort for an infinite rod. Take $O\ o = D\ d = E\ e = \frac{r}{l} \times o\ A'$, when $\frac{r}{l}$ is the ratio of crank to connecting-rod. The points d and e lie on lines parallel to $A'\ B'$. Through the three points pass the arc of the circle. Divide the diameter $A'\ B'$ the same as the indicator diagram, and erect ordinates cutting the circumference in $c_1\ c_2\ c_3$ &c. Draw the horizontals as shown. In the case of the infinite rod, the acceleration efforts are measured from the circumference of the circle to the vertical diameter for the different crank positions, but with the finite rod, the acceleration efforts are measured from the circle to the curved line instead; thus $c_1\ p_1$, $c_2\ p_2 \ldots . c_9\ p_9$, $c_{10}\ p_{10}$ are the acceleration efforts for the crank positions c_1, $c_2 \ldots . c_9$, c_{10}. Transfer the lines $c_1\ p_1$, $c_2\ p_2 \ldots . c_9\ p_9$, $c_{10}\ p_{10}$ with the dividers, as shown in the lower half of the figure, and connect the extremities with the curved line $a\ b$. This line will be the acceleration line required.

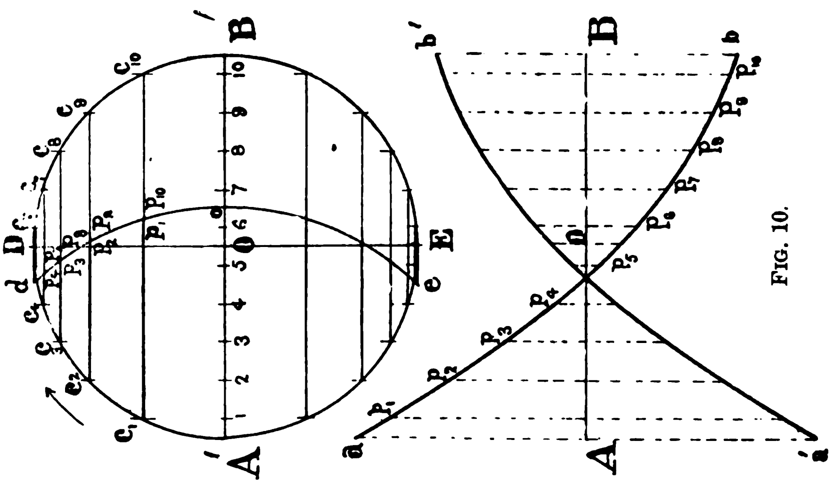
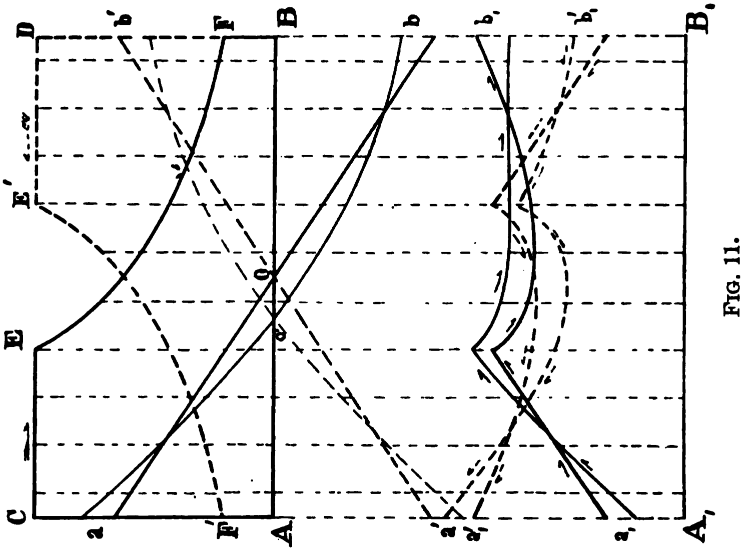

A
B
D
E
O
d
e
c₁
c₂
c₃
c₄
c₈
c₉
c₁₀
e₂
p₁
p₂
p₃
p₄
p₅
p₆
p₇
p₈
p₉
p₁₀
1
2
3
4
5
6
7
8
9
10
a
b
a′
b′
o

FIG. 10.


C
D
E
E′
F
F′
A
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O
a
b
b′
a′
a₁
b₁
A₁
B₁

FIG. 11.

In Fig. 11 indicator diagrams are shown, one in full liens, the other dotted, with the acceleration lines for both the infinite and the finite rod. For convenience, these lines are drawn with reference to the lower line of the diagram. The portions of the diagram between these lines and the upper line of the diagram show the effective pressures against the piston in the two cases. The effect of the finite rod is to lessen the effective pressure on the piston at the beginning and end of one stroke, increasing it in the middle, and vice versa for the return stroke. For still greater clearness the effective pressures are laid off with reference to a base line $A_1 B_1$ where by full lines and by dotted lines they are shown, the arrow heads denoting direction of piston's motion. The difference is very marked.

Fig. 12 shows the effect of acceleration with a short cut-off and high speed. The effective pressures are laid off with reference to $A\ B$. There is a slight pressure driving piston from A to d, a negative effort to pull from d to e, and a positive effort from e to the end of stroke. The changes at d and e sometimes cause a peculiar noise, if the bearings are at all worn, which the engineer is often at a loss to account for. With increased load and longer cut-off the noise disappears. Automatic engines starting without load frequently pound for this reason.

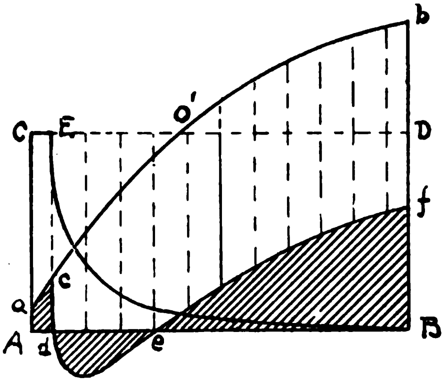

FIG. 12.

Figs. 13, 14, 15 and 16 are diagrams from a "Straight Line Engine" of the following dimensions:*

Diameter of cylinder, 11 ins., stroke, 16 ins., length of connecting-rod, 51 ins., diameter of fly-wheels, 60 ins., face, 8½ ins., weight of fly-wheels, 4000 lbs., weight of piston and rod, 100 lbs, weight of cross-head, 122 lbs., weight of connecting-rod, 150 lbs. Of the weight of the connecting-rod 100 lbs. is considered rotary and balanced in the fly-wheel, leaving 50 lbs. as reciprocating. The total weight of the reciprocating parts is then

$$W = 100 + 122 + 50 = 272 \text{ lbs., whence } w = \frac{W}{A} = \frac{272}{95.03} = 2.86 \text{ lbs.}$$

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* These diagrams were worked up by members of the Senior Class in Mechanical Engineering.

The indicator diagrams in Fig. 13 were taken when the engine was running empty. The revolutions were 240 per minute. *A* is the head end, and *B* the crank end. The spring used was 40 lbs. to the inch, but as the diagrams have been reduced to two-thirds full size the scale is marked 60 lbs. to the inch. The pressure per sq. inch to start the reciprocating parts from rest is, for an infinite rod $= \dfrac{w\,V^2}{g\,r}$.

$$w = 2.86 \text{ lbs.} \quad r = 8 \text{ ins.} = \tfrac{2}{3} \text{ ft.}$$

$$V = \tfrac{240}{60} \times 2 \times \tfrac{2}{3} \times \tfrac{22}{7} = 16.76 \text{ ft. per sec.}$$

$$y = \frac{2.86 \times \overline{16.76}^2}{32.2 \times \tfrac{2}{3}} = 37.51 \text{ lbs.}$$

To this quantity for a rod 51 ins. long and a crank 8 ins. long must be added $\dfrac{8}{51} \times 37.51 = 5.88$ lbs. And $37.51 + 5.88 = 43.39$ lbs., required to start the reciprocating parts from rest · and $37.51 - 5.88 = 31.63$ lbs. to bring them to rest again at the end of the stroke. The pressure for the crank end diagram would be a little greater as the area of the piston-rod decreases the area of the piston; but it may without serious error be considered the same. A circle may now be described with a radius = 37.51 lbs. to a scale of 60 lbs. to the inch, as in Fig. 10, and the points *d*, *o*, *e* found by taking $Oo = Dd = Ee = 5.88$ lbs. to the same scale, and describing the arc of the circle *d o e*. The diameter and circumference being divided the same as the indicator diagram, the horizontals may be drawn and the distances $A'\,o$, $c_1\,p_1$.. ....$C_{10}\,p_{10}$, $B'\,o$ transferred and laid off on the ordinates (not drawn) from the line *C. D.* A comparison of the forward pressure line of the diagram *E*, of the back pressure line of *E'*, and of the acceleration line *a b*, gives us the line of effective piston efforts *r s t u*, referred to the line *A B*. For the return stroke, in the same way, may be found the line *r' s t' u'*. On both strokes the piston efforts become negative at the points *t* and *t'*. The ordinates of the lines *r s t u* and *r' s t' u'* may now be transferred to the lower figure giving the radial diagrams $s\,s_1\,s_6\,O\,s_9\,s_a$ and $s_0\,s_8\,O\,s_2\,s_b$ for the forward and return strokes respectively. From these radial diagrams the tangential diagrams *A' M Q* N B' and *B' N' P M' A'* are found. The mean tangential effort circle may be found by formula, or by averaging ordinates, as already explained. An inspection of the figures shows a *positive* turning effort from *A'* to *N*, and a *negative* turning effort from *N* to *B'*. On the return stroke it is positive from *B'* to *M'* and negative from *M'* to *A'*.

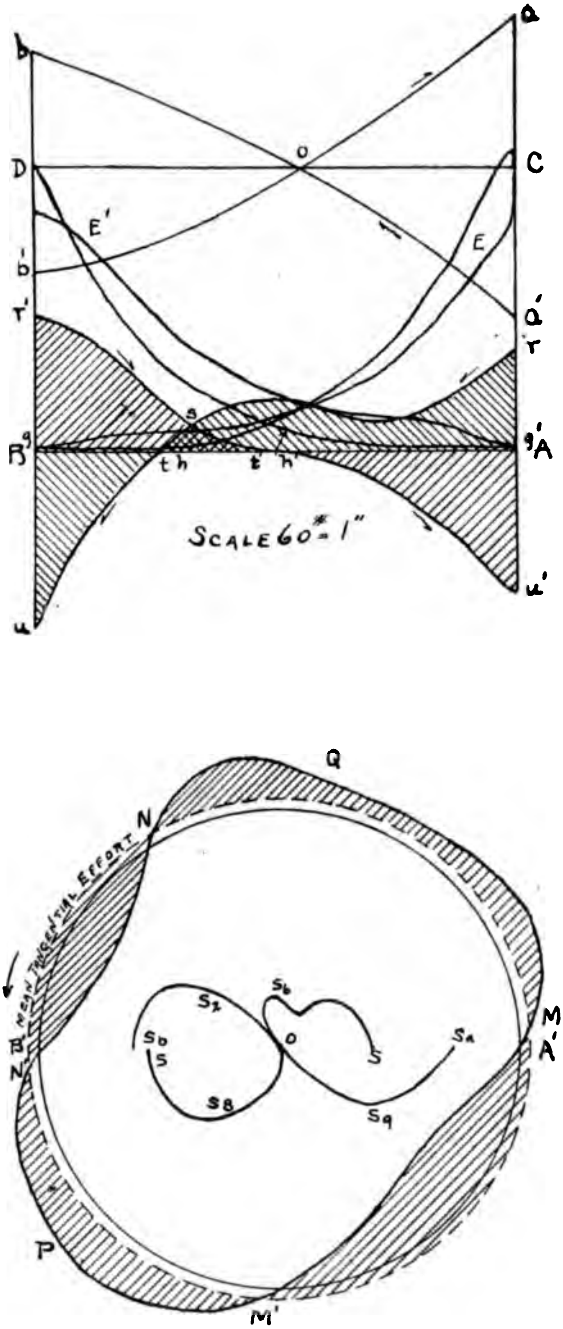

FIG. 14.

HEAD END.

Figs. 14 and 15 show results for the engine under normal load making 235 revolutions per minute. In Fig. 14 the acceleration lines are referred to *A B*, and the lines of effective piston efforts to a line below (not lettered). The radial diagrams of effective piston efforts, and the tangential diagrams in Fig. 15 will be clear from previous constructions.

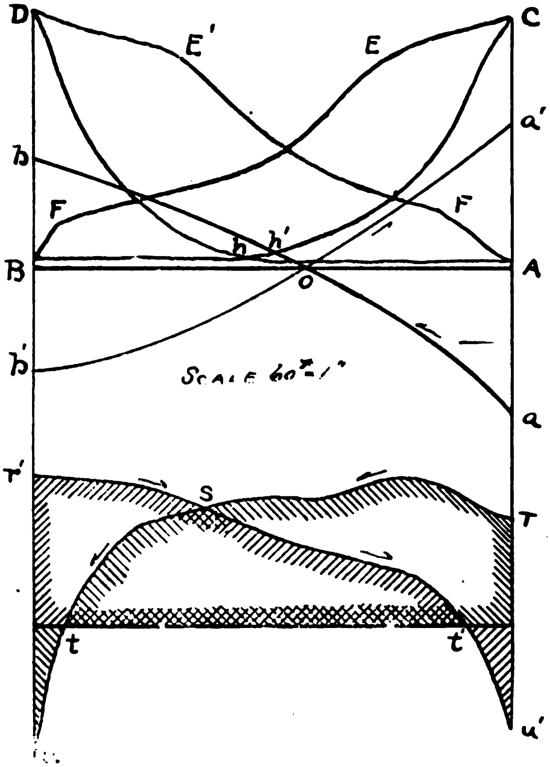

FIG. 15.

The mean effective pressure found by averaging the two indicator diagrams is 26.94 lbs.; and this multiplied by twice the stroke equals the work done in one revolution by one sq. inch of piston area, and it also equals the work done in one revolution on the crank-pin; or

$$26.94 \times 2\,\overline{AB} = p_t\,\pi \times \overline{A'B'}\,;\ \text{whence } p_t = \frac{26.94}{1.57} = 17.16 \text{ lbs.},$$

for the mean tangential effort. This quantity laid off to the scale of 60

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